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Vibration analysis of double-walled carbon nanotubes based on Timoshenko beam theory and wave propagation approach

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Abstract. This paper concerned with the vibration of double walled carbon nanotubes (CNTs) as continuum model based on Timoshenko-beam theory. The vibration solution obtained from Timoshenko-beam theory provides a better presentation of vibration structure of carbon nanotubes. The natural frequencies of double-walled CNTs against half axial wave mode are investigated. The frequency decreases on decreasing the half axial wave mode. The shape of frequency arcs is different for various lengths. It is observed that model has produced lowest results for C-F and highest for C-C. A large parametric study is performed to see the effect of half axial wave mode on frequencies of CNTs. This numerically vibration solution delivers a benchmark results for other techniques. The comparison of present model is exhibited with previous studies and good agreement is found.

Keywords: beams theory; continuum model; micro/nano tubes; natural frequencies; vibrational modes; vibration structure

1. Introduction

Carbon nanotubes (CNTs) fascinate new materials with astonishing mechanical, optical and electrical properties (Ren *et al*. 2011). They are generated by rolling of the graphene sheet (Iijima, 1991). Carbon nanotube sheets include hexagonal cells that are ideally cut to produce carbon atoms of the tube. In fact, CNTs are kinds of rolled graphene sheets, and the rolling manner shows the basic properties of the tube, and that is actually the main reason for the extraordinary feature of the CNTs (Georgantzinos *et al*. 2009). Therefore, in order to effectively use of CNTs in each of these fields, it is important that their vibration characteristics are examined. Owing to the small sizes of the micro beams, they are very appropriate for designing small instruments like sensors and actuators (Subramanian *et al*. 2002).

Hutchison *et al*. (2001) obtained double walled carbon nanotubes (DWCNTs) by arc discharge technique. It was revealed that the inner and outer diameters of DWCNTs are in the range of 1.1-4.2 nm, 1.9-5 nm, respectively with high resolution electron microscopy. Vibration characteristics of SWCNTs and DWCNTs were conducted using flexible shell model (Yan *et al*. 2007). Wildöer *et al*. (1998) and Kwon and Tománek (1998) studied the atomic structure of single-walled carbon nanotubes and multi-walled carbon nanotubes. It is observed that DWCNTs are used to construct the MWCNTs. Li and Chou (2003) used molecular method for the vibrational behavior of CNTs and showed that the results of SWCNTs were 10% higher than those of DWCNTs of the same outer diameter. Ansari *et al*. (2011) engaged Eringen's nonlocal theory based on Rayleigh-Ritz technique to obtain the frequencies of the DWCNT association with different values of ratios and parameters. The results were presented for different zigzag and armchair DWCNTs. Ansari and Arash (2013) used nonlocal model to investigate the vibrations of DWCNTs and the displacement equations are calculated with van der Waals forces. The governing equation for a CNT to study its natural frequencies is given from differential quadrature method (DQM). The mechanical behavior of DWCNTs, with geometrical parameters layer wise boundary conditions and small scale factors is fully investigated. Some new resonant frequencies are introduced to validate the TBM. Yoon *et al*. (2005) utilized frequencies fall in tera-hertz rang using Timoshenko beam model and also used the EBM to find the aspect ratio of DWCNTs. In particular, they compared the significant effect of the Euler and Timoshenko beam models on small diameter and large diameter DWCNTs. They suggested for the vibration of short CNTS, the TBM for tera-hertz is relevant rather than EBM. Rysaeva *et al*. (2020) focused on close packed carbon nanotube bundles materials with highly deformable elements, for which unusual deformation mechanisms. Structural evolution of the zigzag carbon nanotube bundle subjected to biaxial lateral compression with the subsequent shear straining is

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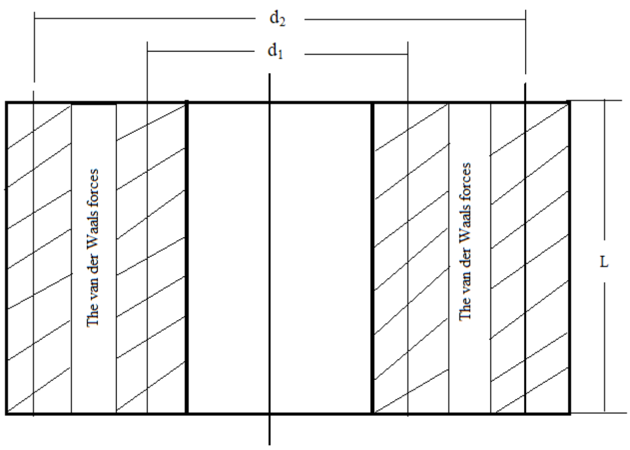

Fig. 1 Schema of double walled carbon nanotubes

studied under plane strain conditions using the chain model with a reduced number of degrees of freedom.

In the present study, a Timoshenko-beam theory is used to investigate the natural frequencies against half axial wave mode with varying four different lengths. DWCNTs are assumed as clamped-clamped and clamped free at both edges. Since some researchers have utilized Timoshenko beam theory, however, do not present satisfactory results about varying half axial wave mode with four different lengths. So these two effects become a significant for vibration frequencies of DWCNTs.

2. Timoshenko-beam theory

Fig. 1 shows the schema of double walled carbon nanotubes based on Timoshenko beam model. According to Yoon *et al*. (2002), the single beam model consists of concentric tubes of multi-walled carbon nanotubes remain coaxial during vibration and the multi beam model having interlayer radial displacements of nested tube within the multi-walled carbon nanotubes. Thus each outer and inner tubes of DWCNTs is treated as Timoshenko beam model. In double walled carbon nanotubes, there are two tubes having diameters d_1 and d_2, respectively and L be the length of the tube. The transverse deflection $w(x,t)$ and the slope $\phi(x,t)$ of a Timoshenko-beam due to bending deformation alone are related by the following two coupled equations (Timoshenko 1974):

$$-GAk\left(\frac{\partial\phi(x,t)}{\partial x}-\frac{\partial^2 w(x,t)}{\partial x^2}\right)+\sigma_x\frac{\partial^2 w(x,t)}{\partial x^2}=\rho A\frac{\partial^2 w(x,t)}{\partial t^2} \tag{1}$$

$$EI\frac{\partial^2\phi(x,t)}{\partial x^2}-GAk\left(\phi-\frac{\partial w(x,t)}{\partial x}\right)=\rho I\frac{\partial^2\phi(x,t)}{\partial t^2} \tag{2}$$

Here x represents the space and t denotes the time variable and $\phi(x,t)$ is the slope of Timoshenko-beam. Where I stand for moment of inertia, A is called area of cross section of CNT and ρ used for mass density of CNTs. σ_x and G represents the distributed pressure and shear modulus. The shear correction coefficient is denoted by k and its value differs for different cross sections such as: thin walled cross section $(0.6\sim0.7)$ and for solid circular cross sections (0.9) (Timoshenko, 1974).

The governing equation of DWCNTs vibration which is gained from Eqs. 1, 2.

$$-GA_1k\left(\frac{\partial\phi_1(x,t)}{\partial x}-\frac{\partial^2 w_1(x,t)}{\partial x^2}\right)+\sigma_x\frac{\partial^2 w_1(x,t)}{\partial x^2}+p_1=\rho A_1\frac{\partial^2 w_1(x,t)}{\partial t^2} \tag{3}$$

$$EI_1\frac{\partial^2\phi_1(x,t)}{\partial x^2}-GA_1k\left(\phi_1-\frac{\partial w_1(x,t)}{\partial x}\right)=\rho I_1\frac{\partial^2\phi_1(x,t)}{\partial t^2} \tag{4}$$

$$-GA_2k\left(\frac{\partial\phi_2(x,t)}{\partial x}-\frac{\partial^2 w_2(x,t)}{\partial x^2}\right)+\sigma_x\frac{\partial^2 w_2(x,t)}{\partial x^2}+p_2=\rho A_2\frac{\partial^2 w_2(x,t)}{\partial t^2} \tag{5}$$

$$EI_2\frac{\partial^2\phi_2(x,t)}{\partial x^2}-GA_2k\left(\phi_2-\frac{\partial w_2(x,t)}{\partial x}\right)=\rho I_2\frac{\partial^2\phi_2(x,t)}{\partial t^2} \tag{6}$$

The pressure on the outer and inner tubes per unit axial length is due to Vander Waals (vdW) forces. The proposed vdW model accounts the deflection of interlayer interactions between the tubes of double-walled CNT. The transverse force applied on the carbon nanotubes is denoted by p. The subscript 1 and 2 are for the designation of outer and inner tubes. The outer and inner layers of DWCNTs are nested with each other and with the help of interlayer spacing; the van der Waals interaction is gained. When these outer and inner tubes deformed coaxially, the interacting pressure with net van der Waals interaction remain zero.

The pressure at any point between the nested tubes is a linear function and can be written as the difference of the deflection at prescribed point.

$$p_1=c(w_2-w_1) \tag{7}$$

$$p_2=-c(w_2-w_1) \tag{8}$$

The term c is the vdW coefficient, depicting the pressure increment contributing from outer and inner tubes and can be estimated erg/cm^2 (Yoon *et al*. 2003).

$$c=\frac{400R_1}{0.16D^2} \tag{9}$$

where R_1 is the inner radius of DWCNTs and $D=0.142\ nm$.

Substitution of Eq. (7) and (8) into Eqs. (3) and (4) leads to 4 coupled equations for 4 unknowns w_j (x, t) and ϕ_j (x, t) $(j = 1, 2)$ which is in the form of wave displacement w

$$w=\{b_1e^{kx}+b_2e^{-kx}+b_3e^{ikx}+b_4e^{-ikx}\}e^{i\omega t} \tag{10}$$

where b_1, b_2, b_3 and b_4 are the constants of integrations and are determined applying the geometric boundary conditions on both ends of the beam.

Table 1 Comparison of frequencies with Ref. (Kumar 2018)

Frequencies (THZ)	Method	*L/d*		
		12	16	20
	DTM	0.32527	0.18298	0.11716
	Present	0.24321	0.13216	0.10928

Table 2 Comparisons of simply supported frequencies with Elishakoff and Pentaras (2009)

L/d	Petrov (Elishakoff and Pentaras 2009)	Present
10	0.46884	0.35561
16	0.18319	0.15421
20	0.11725	0.01245

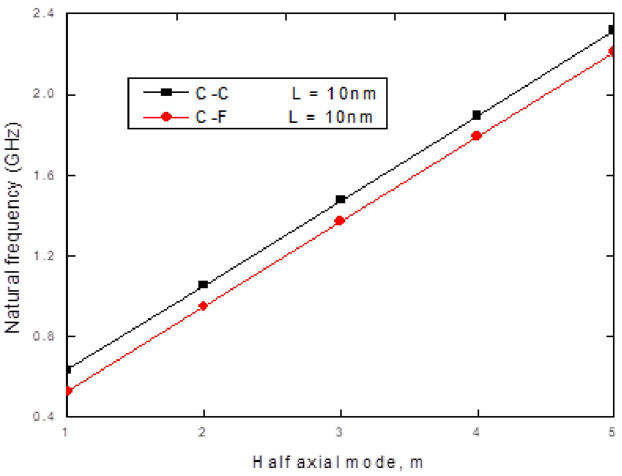

Fig. 2 Half axial wave mode against natural frequencies with length L = 10nm of C-C DWCNT (R_1= 3.5nm, R_2= 5.25nm., G = 0.4, E = 1.0, ρ = 2.0 g/cm^3, σ_x= 0)

2.1 Types of boundary conditions

The continuum approach leads to the fact that the frequency equations for given boundary conditions. Subjecting to different constraints on the displacement function $w(x)$, following geometric boundary conditions are specified.

i. Simply supported boundary conditions:

$w(x) = \frac{\partial^2 w}{\partial x^2} = 0$ at $x = 0$ or $x = L$.

ii. Clamped boundary conditions:

$w(x) = \frac{\partial w}{\partial x} = 0$ at $x = 0$ or $x = L$.

3. Results and discussion

Here, vibrations of double walled carbon nanotubes based on Timoshenko-beam theory is investigated with clamped-clamped (C-C) and clamped-free (C-F) conditions. It is supposed that these two tubes have same Young's modulus (TPa), mass density ($2.3/cm^3$), Poisson ratio (0.25), shear coefficient (0.8), shear modulus (0.4 TPa) and with effective thickness (0.35 nm) (Wang and Varadan, 2005). The proposed model based on Timoshenko-beam

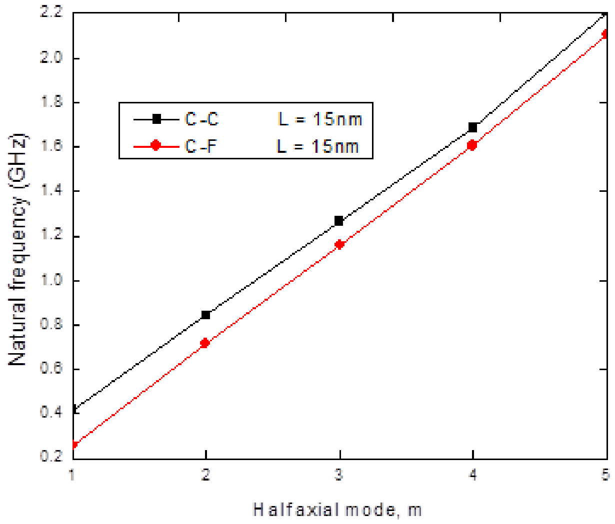

Fig. 3 Half axial wave mode against natural frequencies with length L = 15nm of C-C DWCNT (R_1= 3.5nm, R_2= 5.25nm., G = 0.4, E = 1.0, ρ = 2.0 g/cm^3, σ_x= 0)

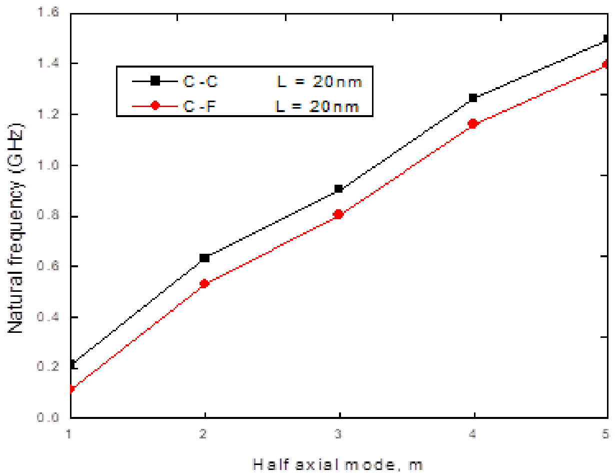

Fig. 4 Half axial wave mode against natural frequencies with length L = 20nm of C-C DWCNT (R_1= 3.5nm, R_2= 5.25nm., G = 0.4, E = 1.0, ρ = 2.0 g/cm^3, σ_x= 0)

model can incorporate in order to accurately predict the acquired results the axial mode m = 1, 2, 3, 4, 5 with lengths L = 10nm, 15nm, 20nm, 25 nm of material data point, E = 1.0, ρ = 2.0 g/cm^3. Table 1 and 2 shows the frequency comparison of carbon nanotubes (CNTs) to account the validity of present computed results. In Table 1, the frequencies are compared with Kumar (2018) and these results were obtained by using DTM. As the aspect ratio (*L/d* = 12, 16, 20) increases, the frequencies (THz) of tube decreases. The present results are little bit from the computations of Kumar (2018). Another comparison is done with the results of Elishakoff and Pentaras (2009) as shown in Table 2. The frequencies for different aspect ratios are consistent for simply supported boundary condition. The convergence and validity of model based on Timoshenko-beam theory is attained with these studies and also verifies that the Timoshenko-beam model can accurately describe the frequency of CNTs with different parameters.

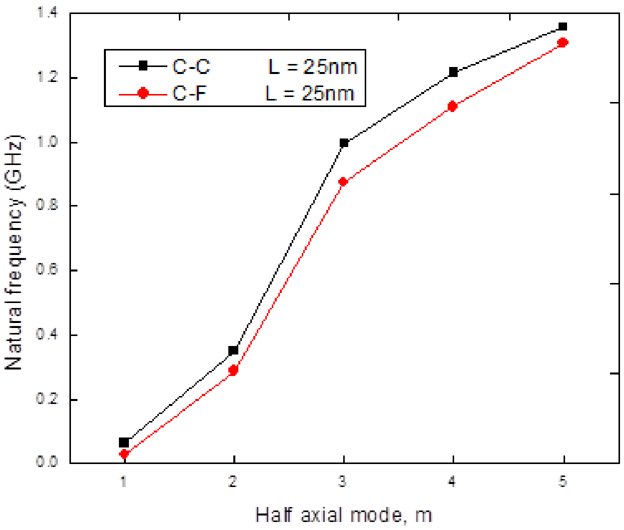

Fig. 5 Half axial wave mode against natural frequencies with length L = 25nm of C-C DWCNT (R_1= 3.5nm , R_2= 5.25nm., G = 0.4, E = 1.0, ρ = 2.0 g/cm^3, σ_x= 0)

Fig. 2 shows the natural frequencies of double-walled CNTs versus half axial wave mode considering the inner and outer radii R_1= 3.5nm, R_2= 5.25nm. The parameters are fixed as G = 0.4, E = 1.0, ρ = 2.0 g/cm^3, σ_x= 0. The length of tube is fixed in this graph as L = 10nm with two boundary conditions clamped-clamped (C-C) and clamped-free (C-F). The effect of frequency is seen for varying the half axial wave mode m (1~5). The frequencies at m =1 are f ~ 0.6319GHz (C-C), f ~ 0.5266 GHz (C-F) and m = 3 are f ~ 1.4744 GHz (C-C) , f ~ 1.3691 GHz (C-F) for fixed L = 10nm. The frequency curve of C-C (L = 10nm) is higher than that of C-F (L = 10nm). The frequencies increase on increasing the half axial wave mode. The display of graph directs that the curves are parallel for both boundary condition and having a minute gape between the curves. Fig. 3 plotted for the frequency effect versus half axial wave mode m with fixed length 15nm for two different boundary conditions. The frequencies at m =1 are f (C-C, C-F) ~ (0.4213, 0.2573 GHz), at m = 2 are (C-C, C-F) ~ (0.8425, 0.7144 GHz), at m = 3 are (C-C, C-F) ~ (1.2638, 1.1586 GHz), at m = 4 are (C-C, C-F) ~ (1.6851, 1.6058 GHz) m = 5 are (C-C, C-F) ~ (2.2063, 2.1061) for fixed L = 15nm. The frequency decreases on decreasing the half axial wave mode. The pattern of frequency curves is different as compared from Fig. 2. In Fig. 2 the frequencies are parallel for all half axial wave modes and have small gap in two boundary conditions but here the frequencies are parallel. The frequency gape is larger for initial value m = 1 and decreases till m = 4 and after m = 4 the frequencies increases abruptly and gape of frequency curve between two boundary condition is in growing form. Fig. 4 exhibit the frequency curves of C-C (L = 20 nm) and C-F (L = 20 nm) versus axial wave mode. The frequencies are presented in GHz. The other parameters are same as in Fig. 2. The frequencies varies as f(m = 1: C-C, C-F) ~ (0.2106, 0.1106 GHz), f(m = 1: C-C, C-F) ~ (0.2106, 0.1106 GHz), f(m = 2: C-C, C-F) ~ (0.6319, 0.5319 GHz), f(m = 3: C-C, C-F) ~ (0.9032, 0.8032 GHz), f(m = 4: C-C, C-F) ~ (1.2644, 1.1644 GHz), f(m = 5: C-C, C-F) ~ (1.4957, 1.3957 GHz). It is seen that the frequency curves are not uniform smooth increase (see Fig. 2) and is not same as Fig. 3. Here the frequency curves are increasing and decreasing behaviors such as fist increases and then decreases, again increases and increases. It means increasing the length of DWCNTs, frequency behavior totally changed. It is seen that model has produced lowest results for C-F and highest for C-C. Fig. 5 present the frequency sketch of frequencies (GHz) versus half axial wave mode with L = 25nm. The C-C frequencies for m = 1, 2, 3, 4, 5 are 0.0643, 0.3502, 0.9943, 1.2148, 1.3564 and C-F frequencies for 0.0286, 0.2873, 0.8744, 1.1086, 1.3058. The frequency pattern is not same as like Figs. 1~3. The complex behavior is observed. The frequency gape is insignificant for m =1 and a bit increase is seen for m = 2, the gap increase for m = 3, 4 and for m = 5, the gap distance same as initial value. It is indicated that the frequencies decreases on increasing the length from L = 10 ~ 25nm.

4. Conclusions

The discussion in this chapter based on Timoshenko beam theory describes the vibration analysis of double walled carbon nanotubes. This vibration behavior is incorporated with clamped-clamped and clamped free boundary condition. The frequencies are estimated with varying half axial wave mode. The frequencies decrease on decreasing the half axial wave mode. At higher values of half axial wave mode, frequencies arcs disturbed. As the length of tube increases with half wave length, the resulting frequencies decreases. The gap between the two distinct boundary conditions is clearly detected especially. The clamped clamped frequencies are higher than other boundary condition. It means that frequencies have a vital role in varying different length of tube. The current model can be extended to other nonlocal model.

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